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Robust impurity-scattering spin Hall effect in a two-dimensional electron gas

V. K. Dugaev,[1,2] M. Inglot,[1] E. Ya. Sherman,[3,4] and J. Barnaś[5,*]

[1]Department of Physics, Rzeszów University of Technology, Powstańców Warszawy 6, 35-959 Rzeszów, Poland
[2]Department of Physics and CFIF, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisbon, Portugal
[3]Department of Physical Chemistry, Universidad del País Vasco UPV-EHU, 48080 Bilbao, Spain
[4]IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain
[5]Institute of Molecular Physics, Polish Academy of Sciences, Smoluchowskiego 17, 60-179 Poznań, Poland

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We propose a mechanism of spin Hall effect in two-dimensional electron gas with spatially random Rashba spin-orbit interaction. The calculations based on the Kubo formalism and kinetic equation show that in contrast to the constant spin-orbit coupling, spin Hall conductivity in the random spin-orbit field is not totally suppressed by the potential impurity scattering. Therefore, the intrinsic spin Hall effect exists being, however, nonuniversal.

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Spin currents are believed to be of great importance for future spin electronics,[1] as they offer the possibility of nonmagnetic manipulation of magnetic moments. Generally, spin currents are associated with charge currents and can be generated by various methods, such as, for instance, by electric field in magnetic systems or circularly polarized light in nonmagnetic semiconductors. Of particular interest, however, are pure spin currents, where the flow of spins is not accompanied by any electric current. Search for generation techniques of pure spin currents, especially in nonmagnetic semiconducting systems, is of high interest both for fundamental and applied physics. One of the possibilities of producing pure spin currents relies on the spin Hall effect (SHE) in nonmagnetic semiconductors with various types of spin-orbit (SO) interaction, where uniform electric field causes a transverse spin rather than charge current.

The existence of SHE in semiconductors with impurities has been predicted by Dyakonov and Perel'.[2] Since then several mechanisms of SHE have been proposed[3–5] and the effect has been observed in a number of experiments.[6,7] It is generally believed that two extrinsic mechanisms related to the SO-dependent scattering by impurities, i.e., side jump and skew scattering,[8–10] can be responsible for the SHE in metallic and semiconducting materials. In addition, a lot of discussions in recent literature concerned the possibility of SHE due to intrinsic SO interaction in disorder-free systems. An extensively studied example of such a system is a two-dimensional electron gas with constant Rashba SO interaction,[5,11] leading to the momentum-dependent spin splitting of electron states. The theoretical efforts were especially focused on the possibility of equilibrium spin currents[12,13] and universal SHE (Ref. 5) independent on the SO coupling strength. However, it turned out that the role of impurities is crucial for this mechanism.[14] It has been shown that even in the limit of a very weak spin-independent disorder, the potential scattering from impurities suppresses the SHE completely.[15–21] In the case of random Rashba field without impurity scattering, the SHE can be also nonzero, as shown numerically within tight-binding model for a finite-size system.[22]

Here we show that the intrinsic SHE does exist, paradoxically, in relatively dirty systems, where the SO coupling appears locally, but vanishes on average. Such a random spin dynamics is common in symmetric semiconductor quantum wells (QWs),[23] such as Si/SiGe (Ref. 24) and GaAs/AlGaAs QWs grown along the [110] axis.[25] Moreover, we show that impurities play here the role less important than in the case of uniform SO interaction—the spin Hall conductivity does not vanish in the limit of small impurity density. This behavior is qualitatively different from that for constant Rashba SO interaction.

To describe the model under consideration we assume Hamiltonian of electrons moving in the $\mathbf{r}=(x,y)$ plane with random Rashba SO interaction, $\hat{H}=\hat{H}_0+\hat{H}_{\rm SO}$ (in the following we use units with $\hbar\equiv 1$) with

$$\hat{H}_0=-\frac{\nabla^2}{2m}, \tag{1}$$

$$\hat{H}_{\rm SO}=-\frac{i}{2}\sigma_x\{\nabla_y,\lambda(\mathbf{r})\}+\frac{i}{2}\sigma_y\{\nabla_x,\lambda(\mathbf{r})\}, \tag{2}$$

where $\nabla_i=\partial_i-eA_i/c$, $\mathbf{A}$ is the vector potential of external field, e and m are the electron charge and effective mass, respectively, and σ_a are the Pauli matrices $(a=x,y,z)$. The curly brackets $\{\cdots\}$ stand for the anticommutator of the appropriate operators to ensure the Hermitian form of the Hamiltonian. The random coupling parameter $\lambda(\mathbf{r})$ vanishes on average, $\langle\lambda(\mathbf{r})\rangle=0$, while the correlator $C_{\lambda\lambda}(\mathbf{r}-\mathbf{r}')\equiv\langle\lambda(\mathbf{r})\lambda(\mathbf{r}')\rangle=\langle\lambda^2\rangle F(\mathbf{r}-\mathbf{r}')$, with all higher correlators reduced to the second-order one for the Gaussian fluctuations of $\lambda(\mathbf{r})$.

The spin current operator has the following form:[26]

$$\hat{j}_i^a=\frac{1}{4e}\{\hat{j}_i,\sigma_a\}, \tag{3}$$

where $\hat{j}_i=-c(\partial\hat{H}/\partial A_i)$ is the ith component of the current operator $(i=x,y,z)$. We consider in-plane electric field $\mathbf{E}$ and calculate the total spin current J_i^a. In the following we use the gauge with vector potential $\mathbf{A}(t)=\mathbf{A}_0e^{-i\omega t}$, $\mathbf{E}=-c^{-1}(\partial\mathbf{A}/\partial t)$, and at the end take the limit $\omega\to 0$ in the calculated response function.[27]

Using Eqs. (1) and (3) one can write the matrix elements of the spin current operator in the basis of eigenfunctions of $\hat{H}_0$ as

$$\langle \overline{\mathbf{k}} | \hat{j}_i^z | \overline{\mathbf{k}'} \rangle = \frac{\delta_{\overline{\mathbf{k}\mathbf{k}'}}}{2m} \left(k_i - \frac{eA_i}{c} \right) \sigma_z, \tag{4}$$

where $\bar{\mathbf{k}}$ includes the electron momentum $\mathbf{k}$ and spin component σ_z. We note that the z component of spin current, $\hat{j}_i^z$, does not contain any anomalous part explicitly dependent on the SO coupling.

It is convenient to decompose the Hamiltonian $\hat{H}$ into two terms, $\hat{H}=\hat{H}_{A=0}+\hat{H}_A$, where $\hat{H}_{A=0}$ corresponds to vanishing vector potential ($\mathbf{A}=\mathbf{0}$) while $\hat{H}_A$ appears at nonzero $\mathbf{A}$. Matrix elements of Hamiltonian $\hat{H}_{A=0}$ are,

$$\langle \overline{\mathbf{k}} | \hat{H}_{A=0} | \overline{\mathbf{k}'} \rangle = \frac{k^2}{2m} \delta_{\overline{\mathbf{k}\mathbf{k}'}} + \hat{V}_{\overline{\mathbf{k}\mathbf{k}'}}, \tag{5}$$

$$\hat{V}_{\overline{\mathbf{k}\mathbf{k}'}} = \frac{\lambda_{\mathbf{k}\mathbf{k}'}}{2} [\sigma_x (k_y + k_y') - \sigma_y (k_x + k_x')], \tag{6}$$

where $\lambda_{\mathbf{k}\mathbf{k}'}$ is the Fourier component of the random Rashba field. In turn, matrix elements of the $\mathbf{A}$-dependent term, $\hat{H}_A$, have the form

$$\langle \overline{\mathbf{k}} | \hat{H}_A | \overline{\mathbf{k}'} \rangle = -\frac{e\mathbf{k}\cdot\mathbf{A}}{mc} \delta_{\overline{\mathbf{k}\mathbf{k}'}} + \frac{e^2A^2}{2mc^2} \delta_{\overline{\mathbf{k}\mathbf{k}'}} + \hat{W}_{\overline{\mathbf{k}\mathbf{k}'}}, \tag{7}$$

$$\hat{W}_{\overline{\mathbf{k}\mathbf{k}'}} = -\frac{e\lambda_{\mathbf{k}\mathbf{k}'}}{c} (\sigma_x A_y - \sigma_y A_x). \tag{8}$$

In the linear-response regime, only the first and third terms on the right-hand side of Eq. (7) are relevant. The third term clearly demonstrates the coupling of electric field to electron spin via the Fourier component of the random Rashba field. These two terms can be associated with two different electromagnetic vertices in the Feynman diagrams for system's conductivity: the first one leads to the conventional conductivity while the third one to the spin conductivity.

To calculate the spin Hall conductivity we apply the conventional Kubo formalism[27] using the retarded and advanced Green's function $\hat{G}_{\mathbf{k}}^{R,A}=\hat{I}G_{\mathbf{k}}^{R,A}$, taken in the vicinity of the Fermi level,

$$G_{\mathbf{k}}^{R,A} = \frac{1}{\varepsilon_{\mathbf{k}} - \varepsilon_F \pm i/2\tau}, \tag{9}$$

where $\hat{I}$ is the 2×2 unit matrix, $\varepsilon_{\mathbf{k}}=k^2/2m$, ε_F is the Fermi energy, and τ is the total momentum relaxation time including scattering from impurities (τ_0) and scattering by spin-dependent Rashba potential (τ_{SO}), $1/\tau=1/\tau_0+1/\tau_{SO}$. Since the SO coupling vanishes on the average, the Green's function, Eq. (9), keeps exactly the diagonal form in the spin subspace. The linear spin Hall conductivity is represented by the sum of two Feynman diagrams in Fig. 1. We neglect ladder corrections for spin current vertex since we assume isotropic scattering by impurities. On the other hand, corrections to the vertices corresponding to the random spin-orbit coupling are small by the parameter $R/\ell \ll 1$, where R is a characteristic length of the fluctuations in the Rashba interaction and ℓ is the electron mean-free path. This situation is completely different from the case of constant SO interaction. Since the spin current operator does not include any anomalous term, there are no diagrams with the corresponding vertices including the random Rashba field. In other words, the spin Hall effect is due to a spin-dependent correction to the distribution function only, as will be explicitly verified later in the text by considering the kinetic equation for the density matrix.[21,28]

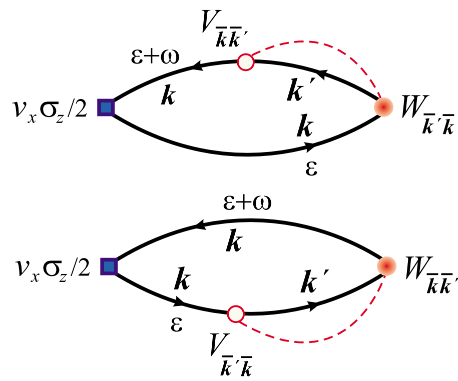

FIG. 1. (Color online) The Feynman diagrams leading to nonvanishing contributions to spin current. Here the left vertex (filled square) corresponds to the spin current operator, the right vertex (filled circle) is the external field perturbation in Eq. (8) and the white circle is the matrix element of spin-orbit coupling in Eq. (6). Upon averaging over disorder, the dashed line becomes the Fourier component of the correlator of random Rashba SO interaction $C(\mathbf{k}-\mathbf{k}')$.

We assume that the field is oriented along the y axis ($A_x=0$, $A_y\neq 0$) and calculate the spin Hall conductivity σ_{sH} defined as $J_x^z=\sigma_{\mathrm{sH}}E_y$. One can easily verify that all other components of the spin current are equal to zero. Calculating the diagrams, taking the trace in the spin subspace, and integrating over the electron energy ε, we obtain in the static limit $\omega\to 0$,

$$\sigma_{\mathrm{sH}} = \frac{ie}{2\pi m} \sum_{\mathbf{kq}} k_x (k_x + k_x') C(q) G_{\mathbf{k}}^R (G_{\mathbf{k-q}}^R - G_{\mathbf{k-q}}^A) G_{\mathbf{k}}^A, \tag{10}$$

where $C(\mathbf{q})=C(q)$ for an isotropic system. We consider the experimentally relevant case of weak SO coupling, where $\tau_0 \ll \tau_{SO}$ and, therefore, τ is very close to τ_0. For the states close to the Fermi surface, the difference $G_{\mathbf{k-q}}^R - G_{\mathbf{k-q}}^A = 2i\,\mathrm{Im}\,G_{\mathbf{k-q}}^R$ can be presented in the form

$$G_{\mathbf{k-q}}^R - G_{\mathbf{k-q}}^A = -2\pi i \delta(\varepsilon_F - \varepsilon_{\mathbf{k-q}}), \tag{11}$$

which reflects the energy conservation. By using the resulting identity,

$$\delta[(k^2 - (\mathbf{k}-\mathbf{q})^2)/2m] = \frac{2m}{q\sqrt{4k^2 - q^2}} [\delta(\theta - \theta_1) + \delta(\theta - \theta_2)], \tag{12}$$

with θ denoting the angle between $\mathbf{k}$ and $\mathbf{q}$, and $\theta_{1,2}$ being two solutions of $\cos\theta_{1,2}=q/2k$, we arrive upon integrating over $\mathbf{k}$ at

$$\sigma_{\text{sH}} = \frac{em\tau}{4\pi^2}\int_0^{2k_F} C(q)\sqrt{4k_F^2 - q^2}dq, \tag{13}$$

where k_F is the Fermi momentum. Taking into account the formula for the spin relaxation time τ_s due to random SO coupling, derived in Ref. 29 for $\ell \gg R$,

$$\frac{1}{\tau_s} = \frac{m}{\pi}\int_0^{2k_F} C(q)\sqrt{4k_F^2 - q^2}dq, \tag{14}$$

we obtain

$$\sigma_{\text{sH}} = \frac{e}{4\pi}\frac{\tau}{\tau_s}. \tag{15}$$

Equation (15) is our main result. It shows that σ_{sH} is nonuniversal and depends on both the disorder due to impurities and random SO coupling. However, it is not zero under very general assumptions of our model, which is qualitatively different from $\sigma_{\text{sH}}=0$ for the uniform Rashba coupling.[18] Of course, our finite σ_{sH} is not in contradiction to the result of Ref. 18 since, in contrast to Ref. 18, the SO coupling is disordered here. Even if the regular contribution, present in real systems, is removed by the vertex corrections, the random contribution in Eq. (15) remains and becomes the leading one.

The above result can also be obtained with the kinetic equation for random Rashba SO interaction,[21,29] which allows a better insight into the problem. The kinetic equation for the density matrix $\hat{\rho}_{\mathbf{k}}$ includes the usual field-dependent term $e\mathbf{E}\cdot\partial\hat{\rho}_{0\mathbf{k}}/\partial\mathbf{k}$, which is responsible for the conductivity. Here the unperturbed density matrix is $\hat{\rho}_{0\mathbf{k}}=\hat{I}f_0(\varepsilon_k)$, where $f_0(\varepsilon)$ is the Fermi-Dirac distribution function. Another source of the perturbation in the electron distribution under external field $\mathbf{E}$ is due to the spin-dependent scattering associated with the fluctuating Rashba SO interaction, as given by the third term in the right-hand side of Eq. (7). As discussed above, the electric field produces a random field acting on electron spin, which can be treated in the collision integral.

Using the matrix elements in Eqs. (6) and (8) and assuming that the perturbation due to SO interaction is small, we find the following expression for the collision integral:

$$\text{St}\hat{\rho}_{\mathbf{k}} = \frac{1}{\tau}(\hat{\rho}_{0\mathbf{k}} - \hat{\rho}_{\mathbf{k}}) + \text{St}^{[E]}\hat{\rho}_{\mathbf{k}}, \tag{16}$$

$$\text{St}^{[E]}\hat{\rho}_{\mathbf{k}} = 2\pi\sum_{\mathbf{k}'}(\overline{\hat{V}_{\mathbf{kk}'}\hat{W}_{\mathbf{k}'\mathbf{k}}} + \overline{\hat{W}_{\mathbf{kk}'}\hat{V}_{\mathbf{k}'\mathbf{k}}}) \times(\hat{\rho}_{0\mathbf{k}'} - \hat{\rho}_{0\mathbf{k}})\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} + \omega), \tag{17}$$

where $\text{St}^{[E]}\hat{\rho}_{\mathbf{k}}$ is the contribution from the random Rashba field. Since we consider the linear response to E_y, in the last term we take the equilibrium density matrix with $\hat{\rho}_{0\mathbf{k}'} - \hat{\rho}_{0\mathbf{k}} = \omega\partial f_0(\varepsilon)/\partial\varepsilon$. Upon the averaging over the SO disorder, we obtain,

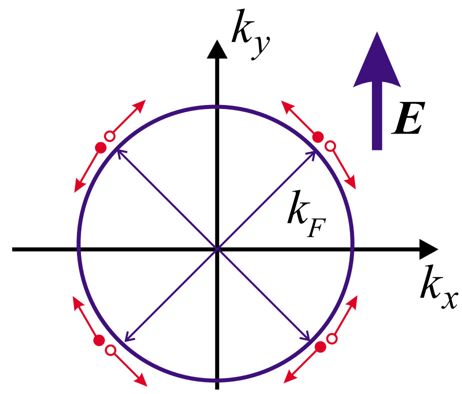

FIG. 2. (Color online) Preferable spin-dependent scattering direction by the effective potential in left-hand side of Eq. (18), shown by arrows attached to the circles. White and filled circles correspond to spin components $\sigma_z=1$ and $\sigma_z=-1$, respectively. As a result, the Fermi line becomes spin split with the preferable concentration of spin-up electrons at $k_x>0$ and spin-down ones at $k_x<0$, leading to the SHE.

$$\overline{\hat{V}_{\mathbf{kk}'}\hat{W}_{\mathbf{k}'\mathbf{k}}} + \overline{\hat{W}_{\mathbf{kk}'}\hat{V}_{\mathbf{k}'\mathbf{k}}} = -\frac{e}{\omega}C(q)(k_x + k_x')\sigma_z E_y. \tag{18}$$

This term is the driving force for the spin current, as shown in Fig. 2. The corresponding contribution to the collision integral can be obtained with Eq. (12) as

$$\text{St}_k^{[E]} = -e\frac{\partial f_0}{\partial\varepsilon}\frac{1}{k}\frac{1}{\tau_s}\left(\frac{k_x}{k}\sigma_z\right)E_y. \tag{19}$$

We present the density matrix as $\hat{\rho}_{\mathbf{k}}=\hat{\rho}_{0\mathbf{k}}+\delta\hat{\rho}_{\mathbf{k}}+S_{\mathbf{k}}\sigma_z$ and with Eqs. (16) and (17), find for the steady state,

$$S_{\mathbf{k}} = -e\frac{\partial f_0}{\partial\varepsilon}\frac{k_x}{k^2}\frac{\tau}{\tau_s}E_y, \tag{20}$$

describing spin split of the Fermi surface, corresponding to Fig. 2. In the stationary state the spin-dependent force due to the external field and random spin-orbit coupling in Eq. (18) is balanced by the friction force due to the disorder, proportional to $1/\tau$.

Having found the distribution function, we can calculate the spin current. First, we calculate in the spin space $\text{Tr}\,\rho_{\mathbf{k}}j_x^z$ with $j_x^z=k_x\sigma_z/2m$ and obtain

$$\text{Tr}\,\rho_{\mathbf{k}}j_x^z = -e\frac{\partial f_0}{\partial\varepsilon}\frac{\tau}{\tau_s}\frac{k_x^2}{mk_F^2}E_y. \tag{21}$$

The total spin current is then equal,

$$J_x^z = \int \text{Tr}\,\rho_{\mathbf{k}}j_x^z\frac{d^2k}{(2\pi)^2} = \frac{e}{4\pi}\frac{\tau}{\tau_s}E_y. \tag{22}$$

This result leads to σ_{sH} equivalent to Eq. (15).

To consider an example, we assume the following form of the correlator $C(q)$:[23,29]

$$C(q) = 2\pi\langle\lambda^2\rangle R^2 e^{-qR}, \tag{23}$$

achieved by doping quantum wells with charged impurities. In the semiclassical limit of long-range correlations, $k_F R \gg 1$, the integral in Eq. (14) becomes

$$\frac{1}{\tau_s} = 2k_F \frac{m}{\pi}\int_0^\infty C(q)dq = 4m\langle\lambda^2\rangle k_F R \tag{24}$$

and the resulting spin Hall conductivity is

$$\sigma_{\rm sH} = \frac{e}{\pi} m\tau\langle\lambda^2\rangle k_F R. \tag{25}$$

When $k_F R \ll 1$, the system is always in the dirty limit of very long SO coupling-determined relaxation times. As a result, the spin Hall conductivity is suppressed and tends to zero as $(k_F R)^2$ with decreasing $k_F R$. This is a general feature of the finite-range correlators [similar to that in Eq. (23)], the effect of which vanishes due to fast oscillations of the Rashba parameter on the spatial scale of the electron wavelength.

Now we can qualitatively discuss the clean limit $\tau_0 \gg \tau_{\rm SO}$. Here the spin conductivity is finite and does not depend on the magnitude of fluctuating Rashba field since both the relaxation rate and gain due to the external field are proportional to $\langle\lambda^2\rangle$. However, the clean limit requires a separate analysis of the relaxation time scales, which will be considered elsewhere.

In conclusion, we have shown that the random Rashba spin-orbit interaction can generate spin Hall effect, even in the presence of impurities. This behavior is distinct from that found for spatially uniform Rashba interaction, where in the limit of small impurity concentration, the potential scattering by impurities totally suppresses the spin Hall effect. In contrast to Ref. 30, where it was found that for the linear Rashba coupling this suppression is a result of a sum rule for spin conductivity, here the corresponding rule cannot be established and the resulting spin current is not suppressed. We mention that the comparison of conventional and torque-related[26] definitions of spin current shows that the result in Eq. (15) is independent of definition.

In systems with nonzero spin polarization, arising, for instance, due to finite magnetization, the above discussed SHE is closely related to the anomalous Hall effect. If the concentrations of spin-up and spin-down electrons are different, spin separation leads to electric current which gives rise to the anomalous Hall effect. Since the disorder in spin-orbit coupling leads to the spin current, it can cause the anomalous Hall effect, too.

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*Permanent address: Department of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland.

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